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The Place of Calculators in Mathematics Education in Developing Countries

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Technology has become a major force in developing curricula and educational practice in mathematics education internationally. While many technologies are important in affluent developed countries, the hand-held calculator continues to be the technology most likely to be available to students when and where they need it. Modern calculators have been designed to support secondary school mathematics education at various levels, including much more powerful ways than merely providing numerical answers to arithmetical questions, and continue to be one of the few technologies designed expressly for educational purposes. In light of economic realities, it is argued that calculators provide the most affordable mechanism for developing countries to provide students with access to technology in mathematics. A major element of calculator use involves individual exploration of mathematical ideas, for which a personal technological device is ideally suited. Support for the work of teachers is recognised as critical for integrating technology into the mathematics curriculum.

Keywords: Calculators; Curriculum; Technology; Developing; Secondary school

Introduction

It is already clear that the developed world is awash with technologies of many kinds, so that it is not surprising to see the extensive ways in which technology has begun to be used in educational settings, with the hope of improving education and ensuring that students are provided with the best possible education for their lives and careers in the 21st century. What is the best way for developing countries to respond to the opportunities provided by new technologies, especially bearing in mind their economic and cultural circumstances?

In the case of mathematics education in particular, the hand-held calculator has become a standard item of equipment for students learning mathematics and teachers teaching mathematics in many countries over the past thirty years. As one of the earliest popular uses of microprocessors, early versions were designed for commerce and industry, not for schools, and were relatively expensive. The last three decades have seen extensive work on refining this invention into a powerful tool for educational purposes, with good connections to mathematics across the spectrum of secondary schooling and the early undergraduate years consciously built into the designs. At the same time, educators and curriculum developers have embraced this technology and adapted curricula, teaching and assessment practices accordingly, while the access costs continue to be much less than for other forms of technology (Kissane, 2007; Ronau et al., 2011).

In this paper, we outline the considerable potential now provided by calculators for mathematics education in developing countries, and argue that they offer the most realistic mechanism for adapting curricula and teaching practices to the new opportunities provided by technology.

Technology in the School Mathematics Curriculum

In recent years, there has been a sharp increase in developed countries in the use of electronic technology generally in mathematics education in schools and in universities. The arguments for such an increase have generally focussed on the potential for technology to improve the teaching and learning of mathematics. For example, a decade ago in the USA, the National Council of Teachers of Mathematics (NCTM) (2000) noted that technology can support, improve and provide new ways of teaching mathematics because technology can focus on the conceptual knowledge before the procedural knowledge is developed. The NCTM captured these ideas in their *technology principle*:

> Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning. … Students can learn more mathematics more deeply with the appropriate use of technology. … In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics. The existence, versatility and power of technology make it possible and necessary to re-examine what mathematics students should learn as well as how best they can learn it. (pp. 24-25)

At the same time, in Australia, the Australian Association of Mathematics Teachers (AAMT) (2000) issued a statement on graphics calculators, suggesting that there was "a compelling case for the advantages offered to students who use graphics calculators when learning mathematics" (p.2), following a conference of teachers who had significant experience in using this sort of technology in schools.

Like these two positions, arguments for the use of technology have mostly centred on improving learning through helping students to engage in personal experimentation and exploration, thus changing the nature of their mathematical experience in various ways. Thus, Hollebrands, Laborde, and Straser (2008) described the new opportunities for learning created by Dynamic Geometry Systems, Friel (2008) explained how the learning of statistics can be changed considerably by access to exemplary software, and Kissane (2009) highlighted ways in which access to the Internet can change the opportunities for learning mathematics in a number of ways.

Recent data suggest that both calculators and computers have become increasingly accepted internationally in many countries. For example, Mullis, Martin, and Foy (2009) reported that about half the countries involved in the large 2007 TIMSS study reported widespread usage of calculators, while "almost all countries permit calculator use for the majority of eighth grade students" (p. 296).

Arguments for the use of technology in school mathematics have also drawn attention to the nature of modern societies, in which access to technology is widespread, and suggested that schools have an important role in making sure that students gain familiarity and confidence with modern tools in order to be useful members of society after they leave school. To illustrate, the draft for the Organisation for Economic Cooperation and Development (OECD) proposed 2012 Program for International Student Assessment (PISA) in mathematics observes:

> The definition of *mathematical literacy* explicitly includes the use of mathematical tools. These tools are physical and digital equipment, software, and calculation devices. Computer-based mathematical tools are in common use in workplaces of the 21st century, and will be increasingly more prevalent as the century progresses. The nature of work-related problems and logical reasoning has expanded with these new opportunities—creating enhanced expectations for *mathematical literacy*. (OECD, 2010; p.7)

The same document suggests that calculators will continue to be used in the PISA assessments, when they are used in the country concerned, and anticipate that a previous policy of being 'calculator-neutral' will be overtaken by a recognition that for some items, calculators will be of assistance to students, noting that "Since PISA items reflect problems that arise in personal, occupational, societal, and scientific contexts, and calculators are used in all of these settings, a calculator is of assistance in some PISA items." (p. 7)

Importantly for the present purpose, however, the capacity of various forms of technology to perform numerical calculations has not been the main argument for their introduction, nor the perceived main benefit. This is so for all forms of technology, including calculators. Arguments for the use of technology in school mathematics, such as those in a recent UK report (Clark-Wilson, Oldknow, & Sutherland, 2011), have focussed instead on the capacity of technologies to help students address situations from a mathematical perspective, and to learn about mathematics in engaging ways, not only to perform numerical computations. In addition, official curriculum documents, such as Section 6.5 of the paper shaping the development of the new Australian Curriculum (Commonwealth of Australia, 2009) acknowledge that technologies have allowed students to tackle everyday problems grounded in the real world, using real data, thus hopefully increasing the relevance of mathematics in the eyes of students. In countries in which calculators have not been routinely used in school mathematics, there has frequently been a perception that the main purpose of the use of calculators is to avoid the need for students to undertake by-hand calculations, with a corresponding unease that students might become mentally lazy or even inept at everyday calculation.

Technology for Developing Countries

The term, 'developing country' is poorly defined, but seems by common usage to generally refer to countries that have limited *economic* resources. Thus, for example, the International Mathematical Union (2012) notes that

> There are vast untapped resources of mathematical talent in the developing world. The desire of students to learn mathematics, and also to become mathematics professionals is growing. At the same time economic resources available to those who want to study advanced mathematics and who have the capacity to excel are woefully inadequate" (p.1).

Such countries may of course be very well developed in other respects, such as culturally and spiritually, so that it is important to recognise the limitations of the term, *developing*. In some cases, developing countries may also lack local leadership and experience, especially in *new* areas like technology use, further exaggerating economic issues.

Limited economic resources are likely to be especially problematic when technology is being considered. At first glance, it might seem that recent efforts to develop inexpensive networked computers might be the most promising avenue to address this problem. However, computer technology is often considerably more expensive than it seems, as Zucker and Light (2009) have observed. They report data for the total cost of ownership for a smart classroom in an urban secondary school in a developing nation, based on using ultra-low-cost computers and the Linux operating system to be around $2000 per seat. Hardware costs account for only around 20% of the costs, which also include training costs, service costs, network costs and various other hidden costs.

While much less sophisticated, and less powerful, than computers, calculators would seem to offer a more accessible alternative for mathematics education in developing countries, at least for the near future. Aside from much lower capital costs, calculators require less 'hidden' costs than do computers, and hence are more likely to be accessible on a wider scale in a developing country. For example, calculators (which are battery-operated) require significantly less infrastructure, such as reliable electricity sources, building cabling, Internet facilities and technical staff, and they are usually easier to transport between classrooms or buildings, as noted some years ago by Bradley, Kemp & Kissane (1994).

In the next section, we explore the variety of kinds of calculators available to schools, and consider their possible contributions to student learning of mathematics.

Levels of Calculators

Since their invention around forty years ago, calculators have been developed and produced at a variety of levels of sophistication. While early calculators were not expressly designed for educational purposes, recent models have been systematically designed and refined for use in mathematics education throughout the school years. In this section, a brief overview of the progression of calculators available today is provided.

Arithmetic Calculators

Inexpensive arithmetic calculators are widely used for everyday business or commercial purposes; a good example is readily seen in most Asian markets, where the calculator also serves as a kind of communication device between buyer and seller. While potentially helpful, especially for young students beginning to learn about the properties of the real number system, such calculators contribute less to learning mathematics than do more sophisticated calculators, because they have the capacity to represent only a very limited range of mathematical ideas.

Scientific Calculators

Scientific calculators have been available for more than thirty years now, and recent models have been refined over the years to meet the needs of secondary school mathematics well. Unlike arithmetic calculators, they can represent numbers in various ways (such as decimals, fractions, and scientific notation) and these days use the same arithmetic logic as conventional mathematics. In addition, following many years of development, modern versions are quite user-friendly and often use syntax and rules of order consistent with mathematical conventions elsewhere.

In addition to representing and using numbers of various kinds, scientific calculators generally include a capacity to evaluate mathematical functions that were previously available only in table books (such as logarithmic, exponential and trigonometric functions) as well as providing access to elementary numerical statistics (such as means and standard deviations), as well as a range of other elementary capabilities (such as evaluating powers and roots, generating random digits, or finding permutations and combinations). Thus, they provide a means for secondary school students to engage personally and efficiently with mathematical ideas that are well beyond arithmetic.

Scientific calculators are sometimes also programmable, thus providing students with an opportunity to explore some aspects of mathematics in new ways.

Advanced Scientific Calculators

In recent years, sophisticated versions of scientific calculators have been developed, offering an extended suite of mathematical capabilities, which in some cases match the extensive suite of capabilities found in many graphics calculators, except for the graphics capabilities. Such calculators provide good numerical support for most of the mathematics of the secondary school curriculum, except those elements requiring images (notably geometry).

Many sophisticated numerical calculations are included. To illustrate, an advanced calculator model provides the following numerical capabilities, among others, in addition to those on typical scientific calculators: equation solving, summation of series, tabulation of functions, matrix arithmetic and inversion, operations with vectors, numerical differentiation and integration, and complex numbers.

Figure 1 shows an example of one of these capabilities which is used to find a numerical solution to the equation $4^x = x + 2$. In this case, the calculator has located an approximate value for x for which the equation is true, and provided some verification for this by evaluating the numerical difference between the expressions on each side of the equals sign.

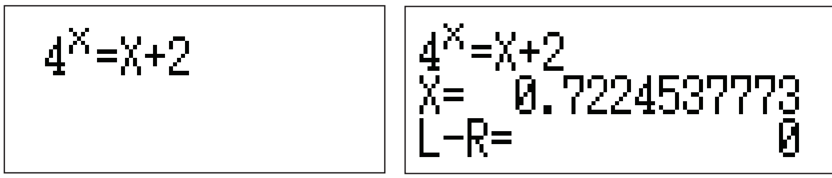

Figure 1. Numerical equation solution.

As its name suggests, a calculator may be used to undertake calculations; in addition, however, access to mathematical capabilities of calculators provides students with new opportunities to see for themselves important fundamental relationships between mathematical ideas. Figure 2 shows an example of this which is concerned with the definition of logarithms and the relationship between logarithms and exponents.

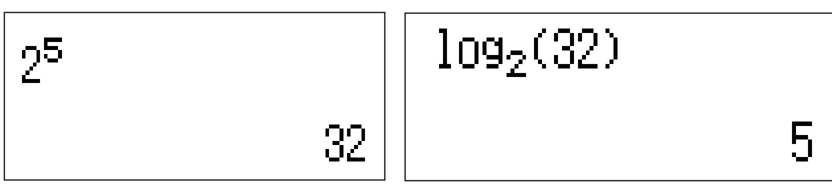

Figure 2. Exploring a mathematical relationship.

Advanced scientific calculators are related to many aspects of the secondary school curriculum. To illustrate this, Figure 3 shows some examples of numerical capabilities of the same calculator to evaluate an integral, perform some complex arithmetic and find the inverse of a matrix.

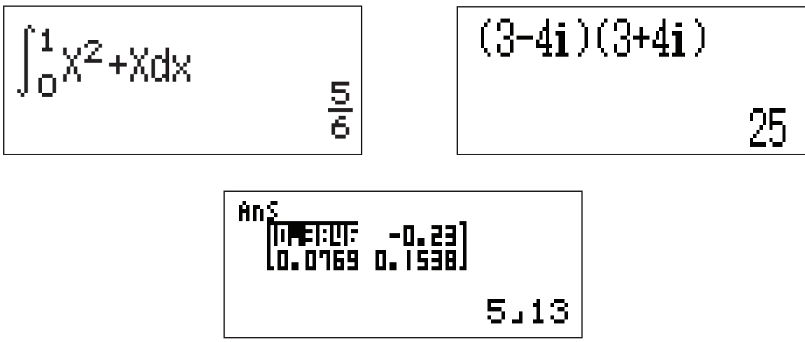

Figure 3. Some other numerical capabilities of an advanced scientific calculator.

By conscious design, advanced scientific calculators offer students and teachers opportunities to engage with a wide range of mathematics relevant to the secondary school, especially in the senior years of schooling or the early undergraduate years. Many examples of suitable classroom activities are freely available on the Internet (e.g. CASIO Worldwide Educational Website, 2012).

Graphics Calculators

The next level of sophistication comprises graphics calculators, which typically include capabilities of advanced scientific calculators but also have a graphics screen, so that a wider range of aspects of mathematics can be represented and manipulated by the calculator user, and especially by students.

The aspects of mathematics that become accessible with a graphics calculator include many elements that are commonly encountered in secondary school. These include graphs of functions, displays of statistical data, probability distributions, geometric objects, conic sections and definite integrals. To illustrate some of these briefly, Figure 4 shows how a graphics calculator can represent a function in three different ways (using symbols, a graph or a table) for students to explore, while Figure 5 illustrates the use of a calculator to represent a probability distribution, manipulate geometric objects or explore the convergence of a series.

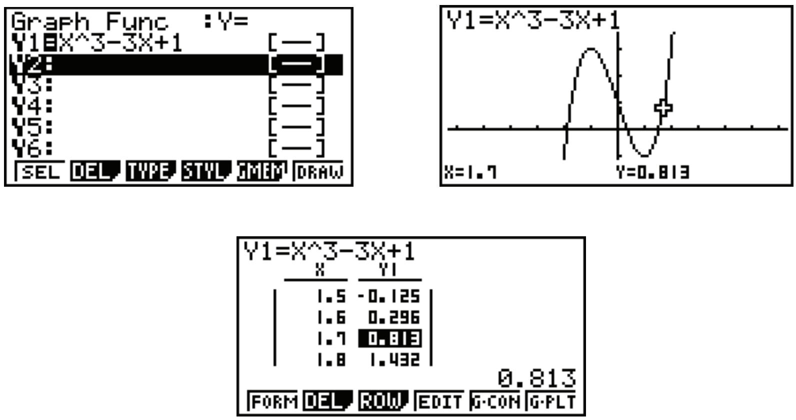

Figure 4. Three representations of a function on a graphics calculator.

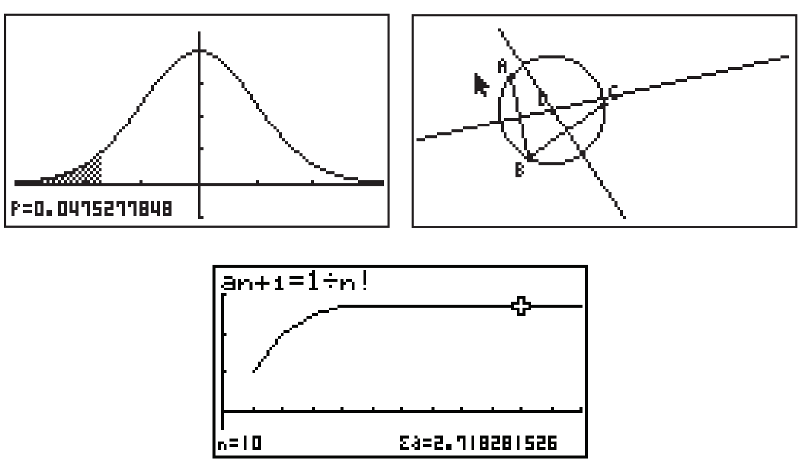

Figure 5. Using a graphics calculator to explore aspects of mathematics.

Graphics calculators have been widely used in developed countries for almost two decades now, having first appeared in the mid-1980s, so that modern versions have been regularly refined to take advantage of the experiences of students and teachers over a considerable period of time, and provide powerful support for many aspects of secondary school mathematics. Educational materials of various kinds have been produced over time as well, providing extensive help for both students and teachers; a good example is Kissane and Kemp (2006).

Unlike the original scientific calculators of more than 30 years ago, graphics calculators are learning tools that have been designed and refined expressly to suit the needs of secondary school students learning mathematics. Thus they are not necessarily tools intended for sophisticated adults, and are not designed to be used by advanced professionals, such as engineers, scientists and applied mathematicians, who are more likely to need sophisticated computer software to meet their needs. As the AAMT communiqué (2000) noted, and Kissane (2007) elaborated in some detail, graphics calculators show considerable promise in supporting the learning and teaching of mathematics in the secondary school.

Of course, not all graphics calculators are the same, with different manufacturers providing slightly different capabilities and operating systems, and the same manufacturers developing a variety of models for particular purposes. There are relatively unsophisticated models available, with reduced capabilities and price, and relatively sophisticated models available, including several with Computer Algebra Systems (CAS) included. Some recent models also include colour capabilities and larger, high-resolution displays.

The relevance of graphics calculators to developing countries has been sharply affected by the price of the calculators to students, which has in turn affected the likelihood that the calculators have been used sufficiently widely to be considered by curriculum development agencies, such as Ministries of Education. There would seem to be a good case for a close look at the potentials of this technology, including less sophisticated and thus less expensive versions of it, for developing countries.

How Should Calculators Be Used?

Hand-held calculators are essentially personal technologies, and a major advantage of their small size and portability is that students can readily take them from place to place (such as class to class or between school and home). To exploit this technology well, ideally students should be able to use calculators often, rather than in a very restricted way (such as that common for computer laboratories).

The strongest educational arguments made for the use of calculators are that they offer students new ways to explore mathematical ideas for themselves; these ideas are not restricted to numerical ideas, of course. So, it is a measure of the limited progress made internationally in this regard that the most recent TIMSS data (Mullis, Martin, & Foy, 2009) do not suggest that calculators are well-used yet, even in those countries that use them:

> On average internationally, teachers asked the greatest percentages of students using calculators in solving complex problems (31%), checking answers (26%) and doing routine computations (25%). Only 16 percent, on average, were asked to explore number concepts. (pp. 296-297)

While higher percentages associated with exploring mathematical ideas might be expected for older students, for whom other aspects of mathematics such as algebra are more significant, it seems clear that the availability of a calculator does not by itself mean that productive use will be made of it. Allowing students to encounter and explore by themselves mathematical ideas and their relationships requires that the calculators are available, that students know how to use them, that the teacher stimulates productive activity and that the school curriculum supports work of these kinds.

Realistic applications of mathematics can be tackled if a calculator is available, so that students can use data they have obtained themselves and other kinds of real information, rather than relying on sources further from their immediate world (such as textbook examples). Armed with a calculator, students will never encounter computations beyond their capabilities to complete, which might encourage and help them see the relevance of mathematics to the everyday world.

The Importance of Teachers

Many researchers have noted that teachers are central to the intelligent use of calculators in schools, and thus will need well-designed support to ensure that calculators are appropriately used. Some of this support will take the form of professional development of various kinds, but it will also be necessary for materials to be developed to suit the needs of teachers in classrooms.

As suggested by Kissane (2003), teachers need to be confident users of technology themselves before they will effectively help their students, so that the early phases of professional development for teachers might need to concentrate on the development of expertise with a calculator, before expecting that teachers can take the adventurous step of using them in a classroom. A second step might involve the production of exemplary materials, well tried in local schools and found to suit the local curriculum, to assist teachers to help students in their class make good use of calculators for learning. Only later, will it be realistic to expect teachers to begin to adapt the calculator to suit their curriculum, needs and interests, and to start contributing to a wider process of curriculum change.

In developing countries, there may be special challenges in providing support for teachers, including limited resources, the availability of time for teachers, geographical problems and the use of languages that are accessible to all (since many existing materials have been developed in English, rather than local languages). Professional development models that concentrate on developing local expertise to help local teachers and involve carefully adapting foreign ideas to local conditions may be worthwhile and manageable. For many teachers, the most helpful partners in developing expertise will be fellow teachers at the same school, at least at first, so that so-called ‘train the trainer’ models may be especially important.

The use of calculators in the most powerful ways, focusing on developing mathematical concepts and not just on calculation, may require significant shifts of pedagogical approaches for some teachers in some developing countries. An example of this kind of issue was the observation by a UN report regarding curricula and teaching practices in some Arab countries:

> The Report contends that curricula in Arab countries have been reinforcing submission, obedience and compliance at the expense of creativity and critical thinking. The Report team stresses the need for an urgent shift from emphasis on rote learning and memorization, which have stifled the creativity of Arab students, to greater emphasis on critical thinking, in line with international trends in mathematics and science. (United Nations Development Programme, 2007)

Using a calculator to learn with, and to learn from, rather than regarding a calculator as merely a device to provide numerical answers to numerical questions requires a shift of pedagogical stance, which ought to be recognised as difficult for some teachers, and which will require some specific attention and support to accomplish.

Technology and the Mathematics Curriculum

As suggested earlier by the NCTM (2000) statement, access to technology allows for and requires a revised mathematics curriculum. However, it is usually difficult to implement new curricula, especially if they require significant investments of time and other resources. For developing countries, changing a curriculum to take advantage of new technologies is a risky enterprise, unless there is a reasonable assurance that the technologies concerned will actually be available and appropriately used in schools. In fact, the situation is no different in developed countries, as argued by Kissane (2002).

For clear economic reasons, it seems more likely that less expensive technologies such as calculators will be available on a wide scale than will computers, for most developing countries. If calculators are assured to be widely available, then the mathematics curriculum can be confidently changed; this includes examinations. Without official curriculum sanction, however, it is unlikely that many schools will acquire adequate resources or have realistic incentives to develop the necessary expertise among mathematics teachers.

Possible changes to curricula include: an increased emphasis on numerical solution to equations generally, rather than a reliance on exact solutions for a small number of kinds of equations; use of reducible interest (which is what occurs in practice) rather than simple and compound interest (which usually don't occur in practice); numerical rather than analytic approaches to calculus problems such as finding relative extrema or numerical solutions to differential equations; changing the emphasis in statistics from mathematical statistics to data analysis, using real data and real problems.

If mathematics curricula are changed to acknowledge and exploit the use of calculators, then textbooks and other materials also need to be adapted, in due course, to ensure that both students and teachers have a good understanding of changed expectations as well as the fresh opportunities involved.

Conclusion

The hand-held calculator has been refined by manufacturers and adapted by educators to suit the needs of the school mathematics curriculum. In addition to providing a mechanism for producing answers to numerical questions, calculators also offer an opportunity for students to learn about mathematics in new ways, and for teachers to adapt their instruction to encourage and permit students to engage in productive activity. Bearing in mind the limited economic resources available in developing countries, and the many needs that have to be met, calculators offer the most appropriate path to take advantage of technology to improve mathematics education for all students, not just for a relatively affluent minority. To take advantage of this technological development, the work of teachers needs to be adequately supported, and suitable changes to the school mathematics curriculum are needed. There are many examples of these sorts of curriculum adaptations available in developed countries, and these are likely to be helpful to the process of taking technology into account in the mathematics curriculum of developing countries.

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